




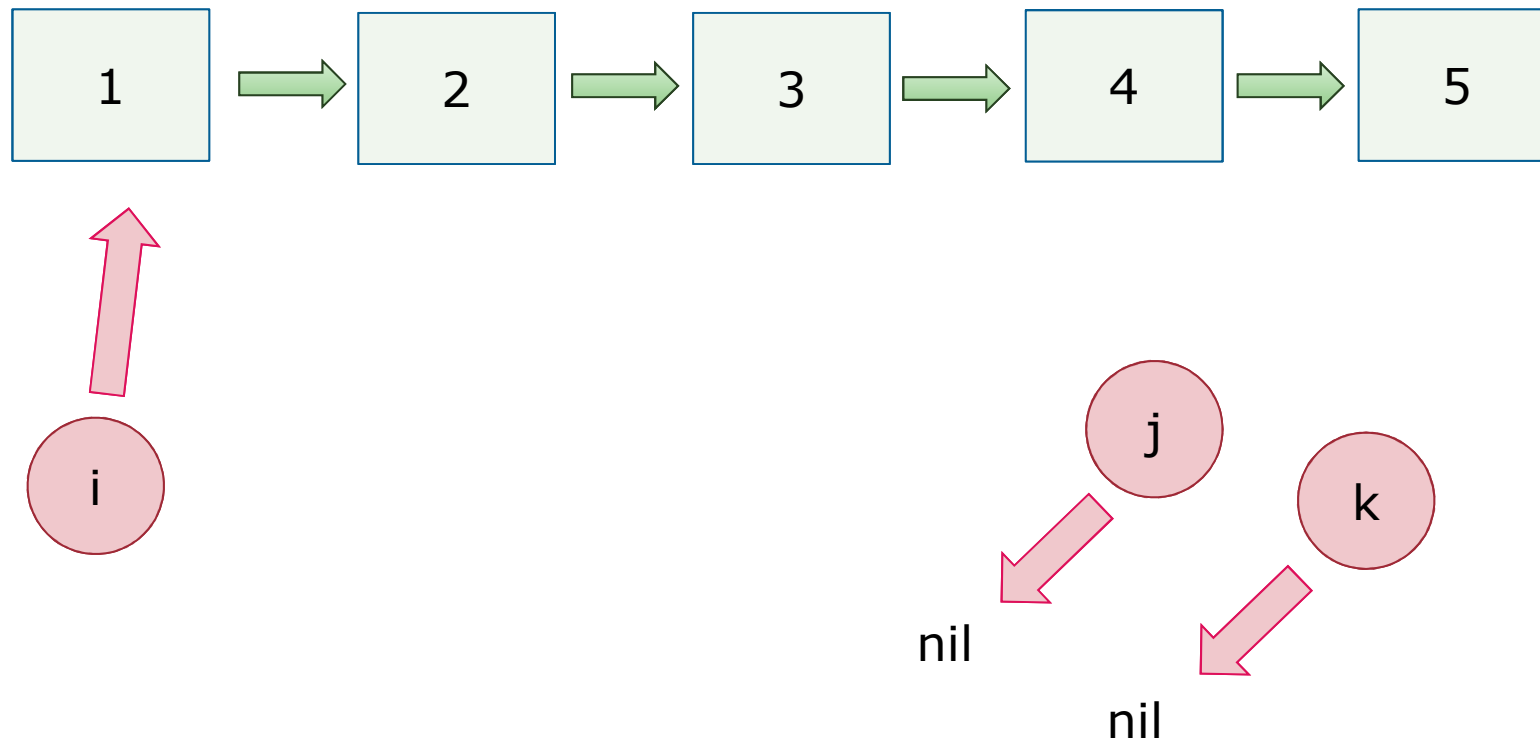
# Separation as Abstraction

Nisansala Yatapanage  
with Cliff Jones and Andrius Velykis  
Newcastle University

- 
- Issues of separation are well handled by Concurrent Separation Logic.
- 
- (at least) Some examples can be clearly developed using layers of abstraction.
- 
- Two examples: Reynolds' in-place list reversal algorithm and concurrent DOM trees.

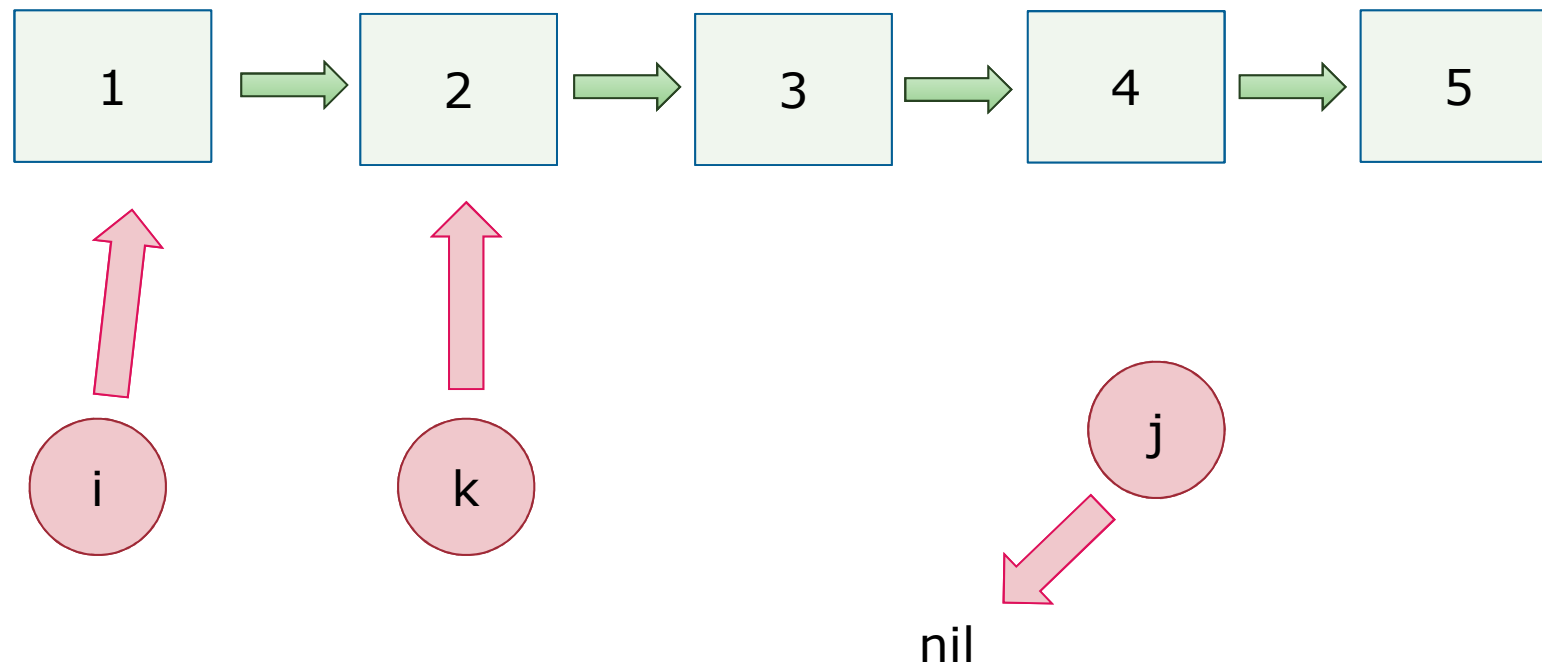
# In-place List Reversal

Originally presented by John Reynolds



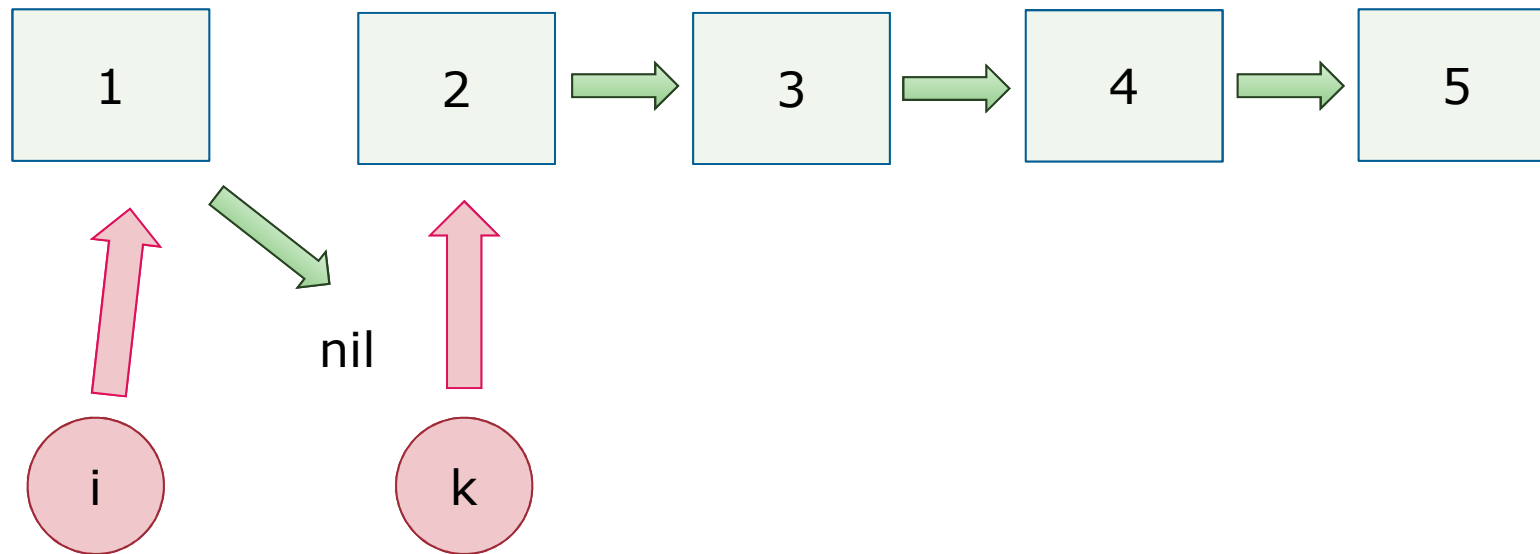
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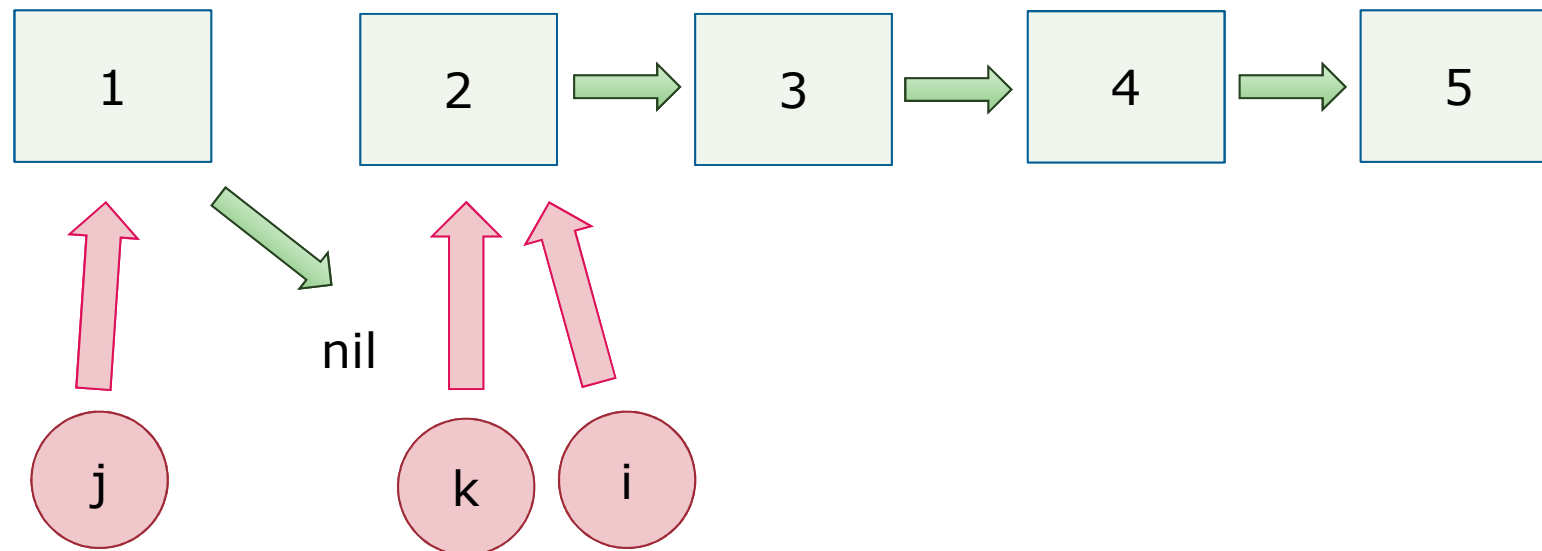
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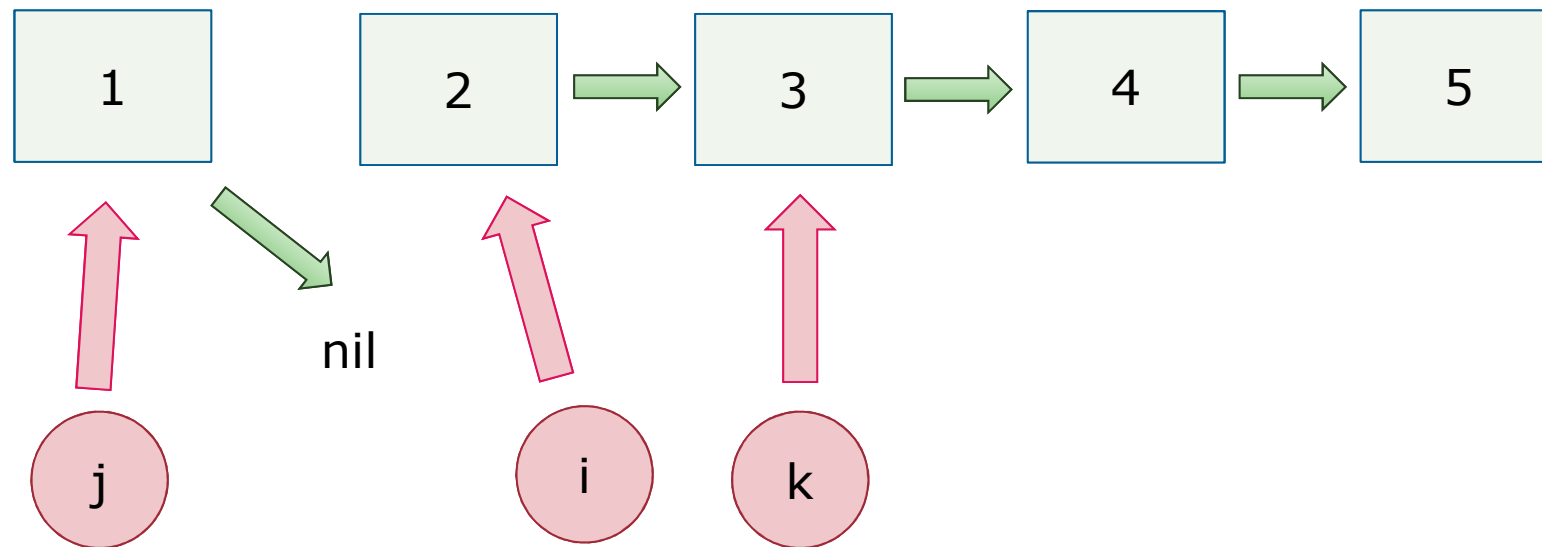
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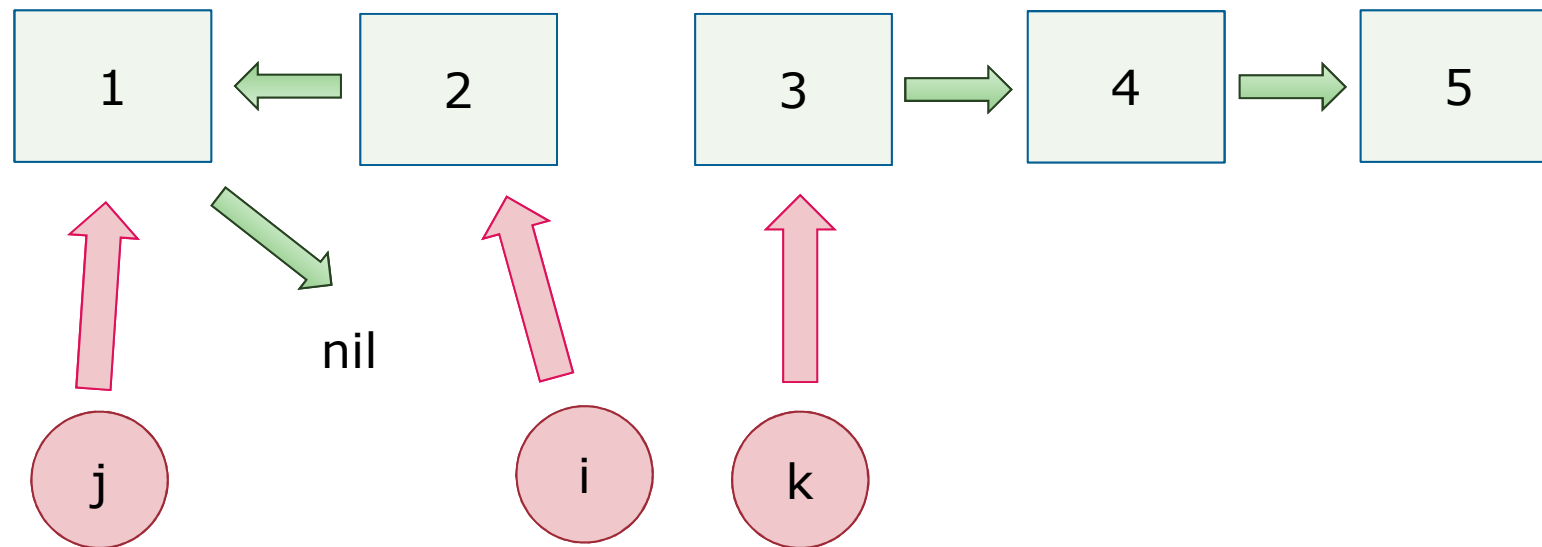
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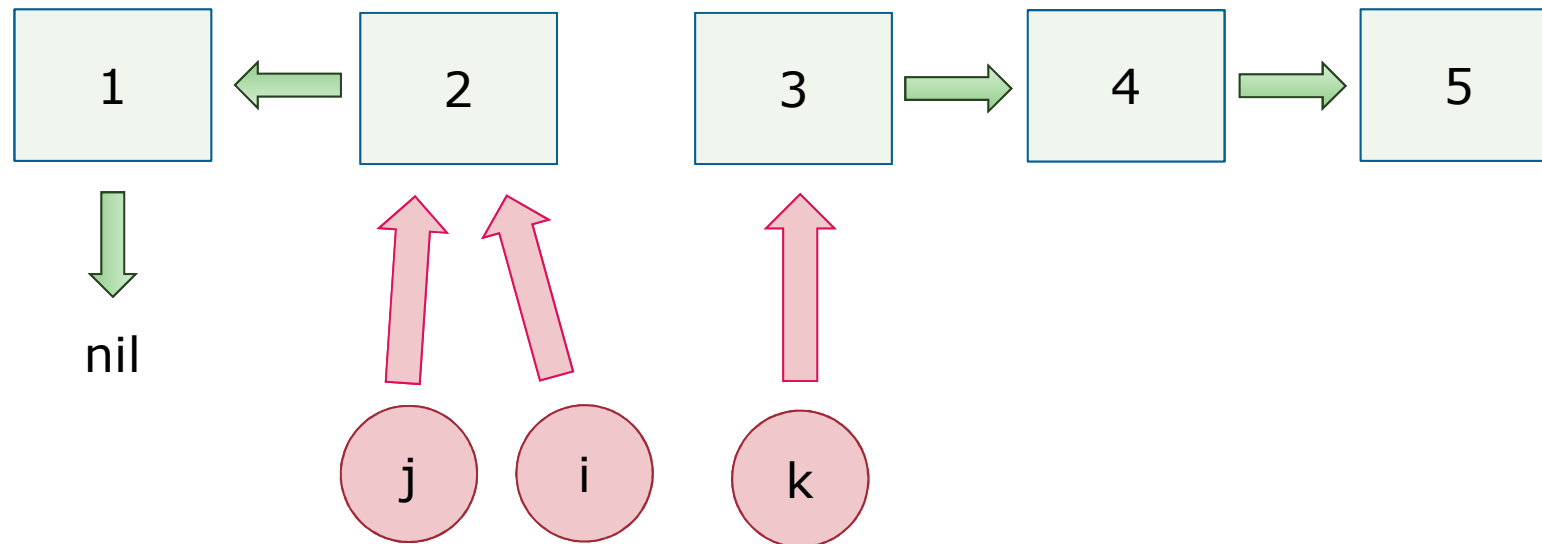
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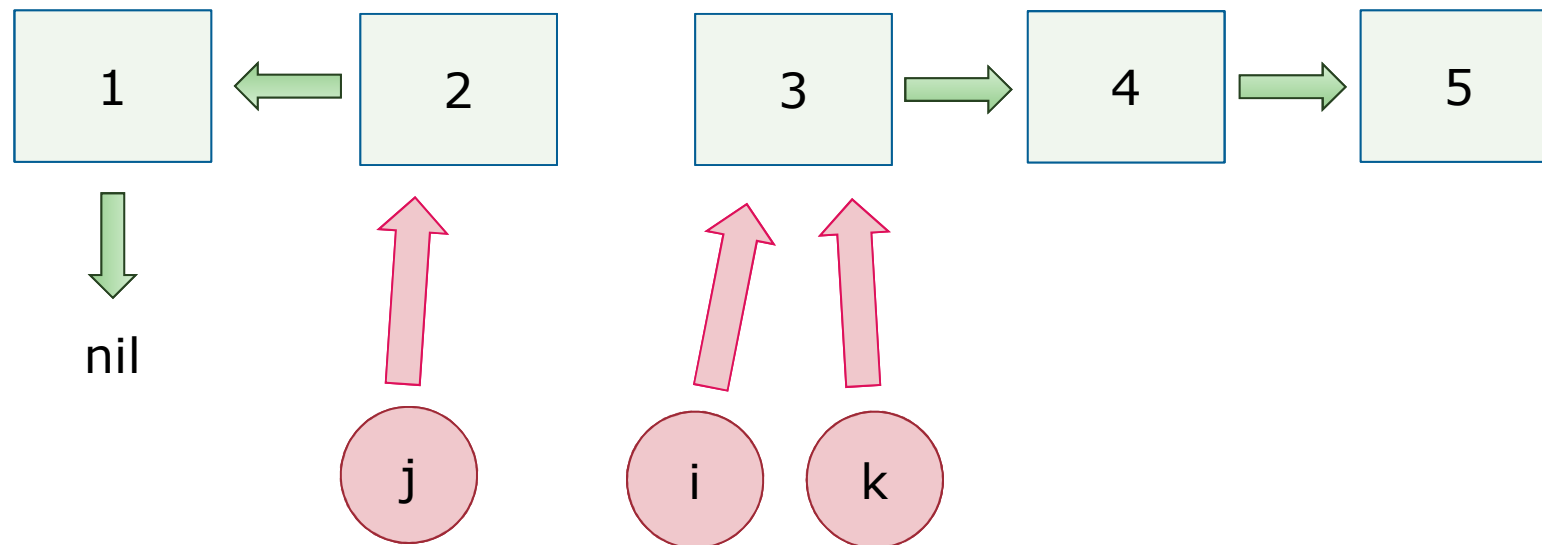
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# Abstract List Reversal

```
while s is not empty do  
     $r' = \text{head of } s \text{ joined to } r$   
     $s' = \text{tail of } s$   
end while
```

Postcondition:  $r' = \text{rev}(s)$

# Separation

- In the abstract specification,  $r$  and  $s$  are assumed to be distinct.
- In normal data reification, the separation is preserved.

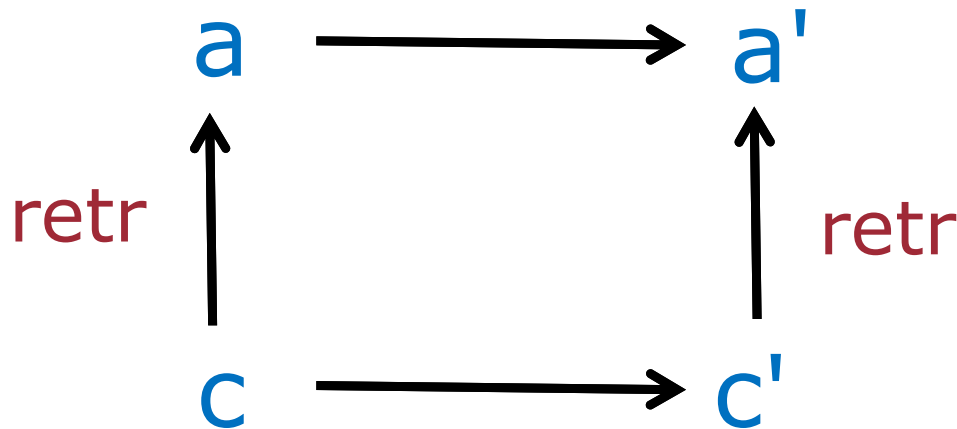
# Separation

- In the abstract specification,  $r$  and  $s$  are assumed to be distinct.
- In normal data reification, the separation is preserved.
- For Reynold's algorithm, we are reifying  $r$  and  $s$  onto parts of the same vector.
- This leads to a new form of reification: preservation of separation.

# Implementation

The separation has to be stated as a predicate in the invariant.

The implementation can be shown to satisfy the abstract specification.



## Proof – retr

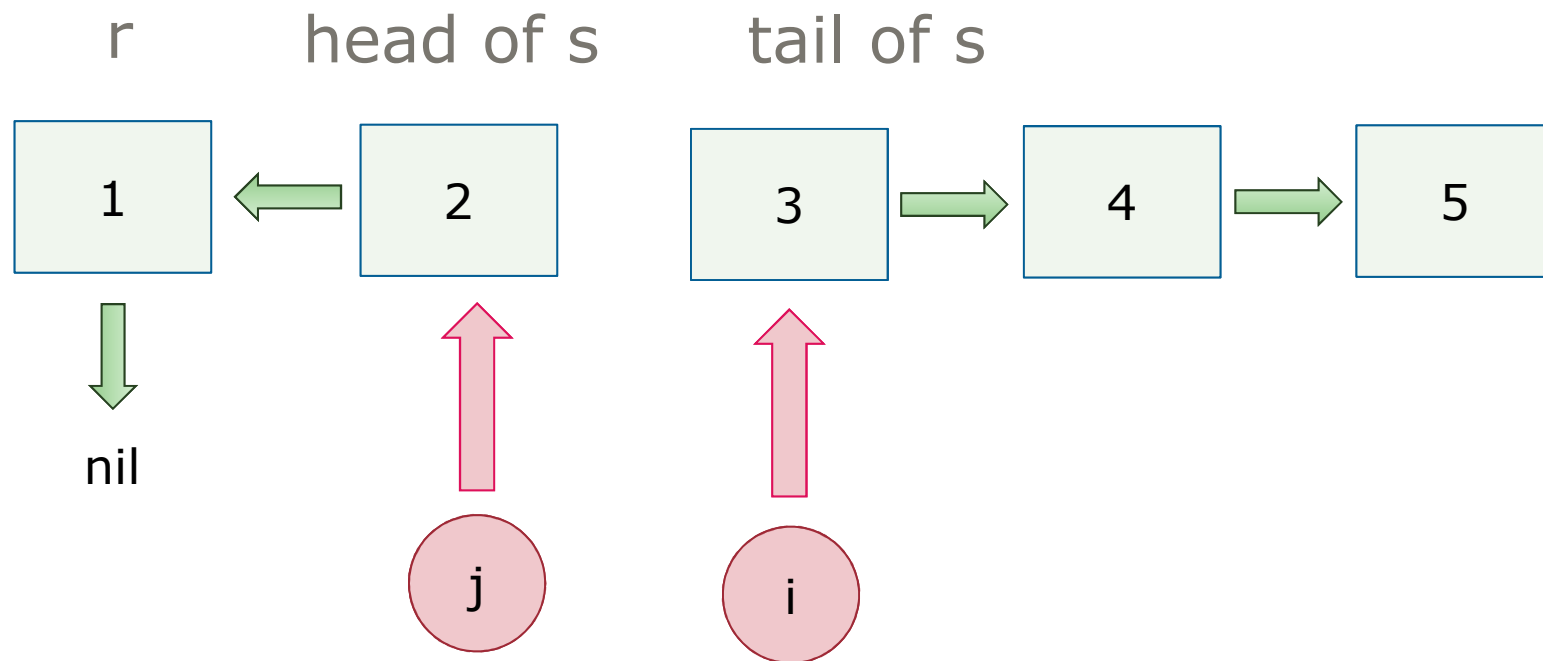
$$\text{retr} : \Sigma_r \rightarrow \Sigma_a$$

$$\text{retr}(mk\text{-}\Sigma_r(m, i, j)) \triangleq \\ mk\text{-}\Sigma_a(\text{gather}(i, m), \text{gather}(j, m))$$

$$\text{gather}(n, m) \triangleq \\ \text{if } n = \text{nil} \\ \text{then } [] \\ \text{else let } (v, p) = m(n) \text{ in} \\ [v] \curvearrowright \text{gather}(p, m)$$

# Proof

At each step, recall:  $r' = \text{head of } s \text{ joined to } r$ ;  
 $s' = \text{tail of } s$





# Concurrent DOM Trees

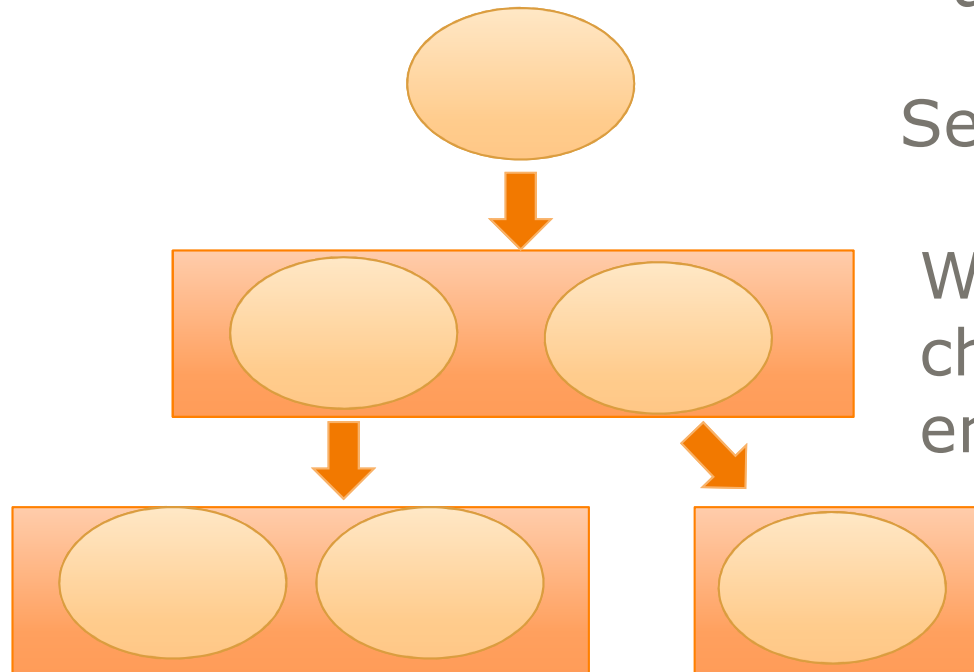
- Philippa Gardner & colleagues published proofs using separation logic of sequential algorithms for updating DOM trees.

# Concurrent DOM Trees

- Philippa Gardner & colleagues published proofs using separation logic of sequential algorithms for updating DOM trees.
- Our first attempt at abstraction used recursive structures.
- This wasn't suitable as we needed NodeID's to match with DOM.

# Concurrent DOM Trees

## Abstract trees



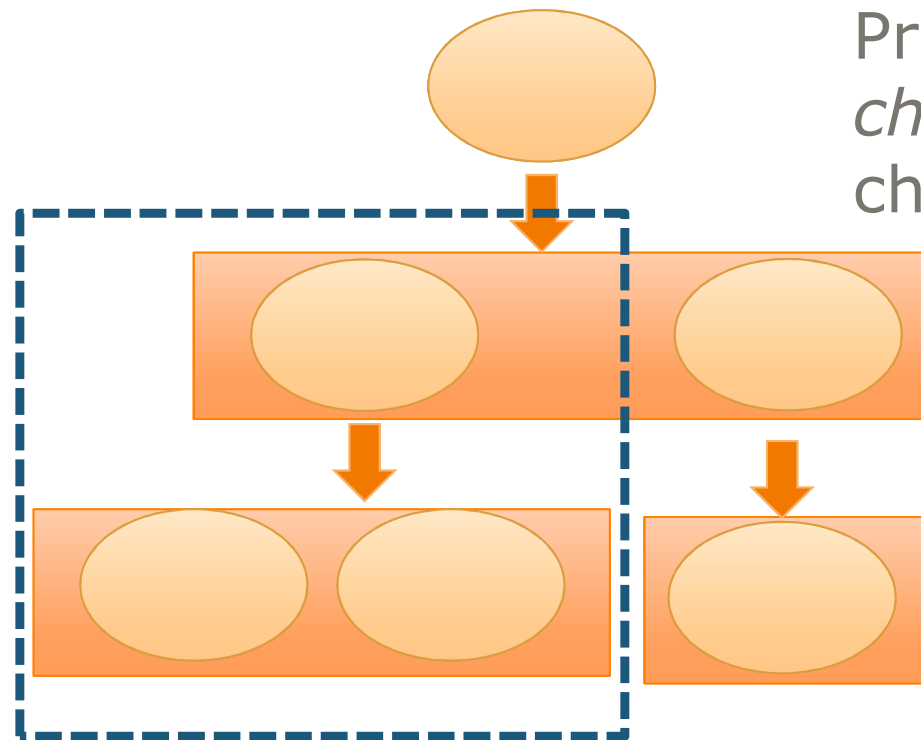
Each node has data and a list of children.

Separation is implied by:

Well-foundedness of the child to parent relation, ensuring no loops.

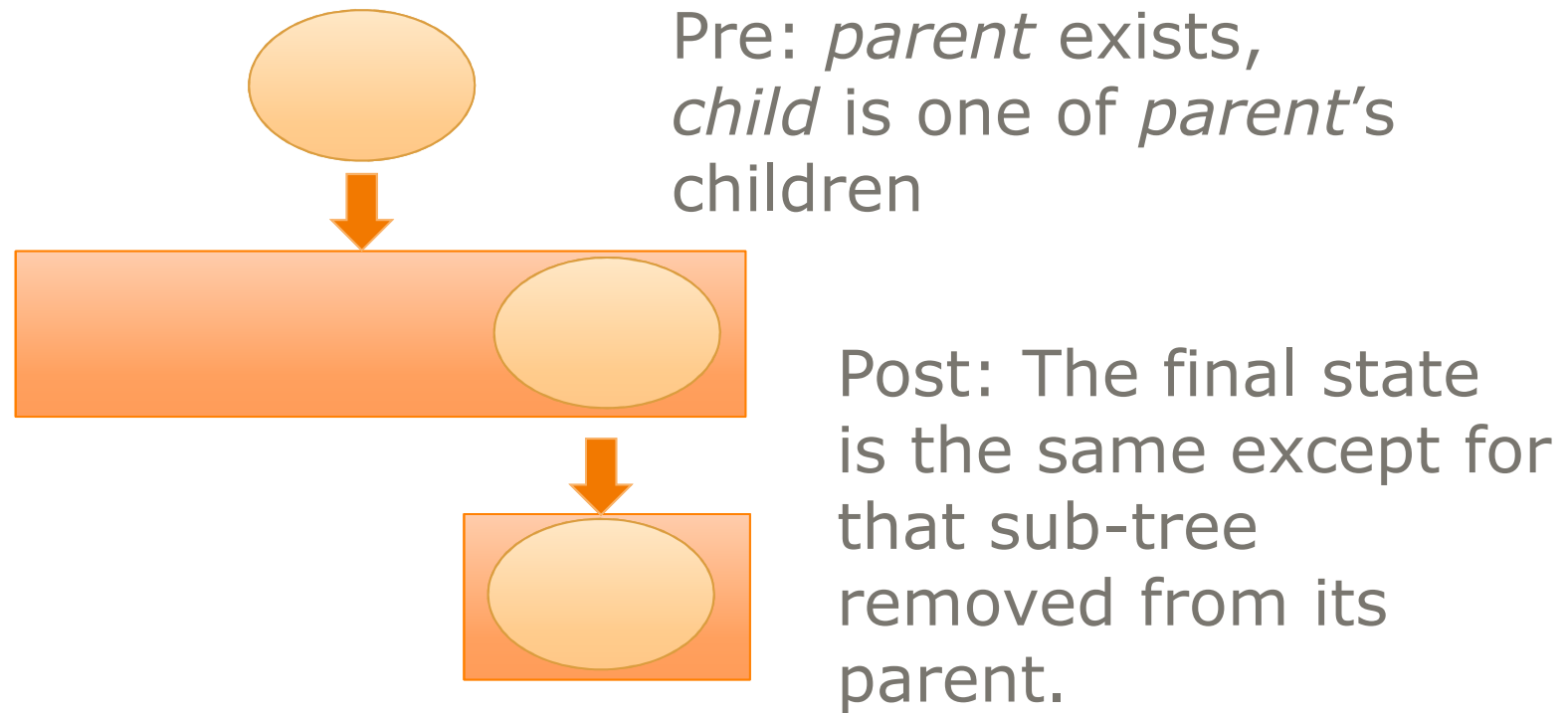
No child has more than one parent.

# Abstract Sequential Remove



Pre: *parent* exists,  
*child* is one of *parent's*  
children



# Abstract Sequential Remove



# From sequential to concurrent

-  The sequential postcondition can state equalities about the whole system.

# From sequential to concurrent

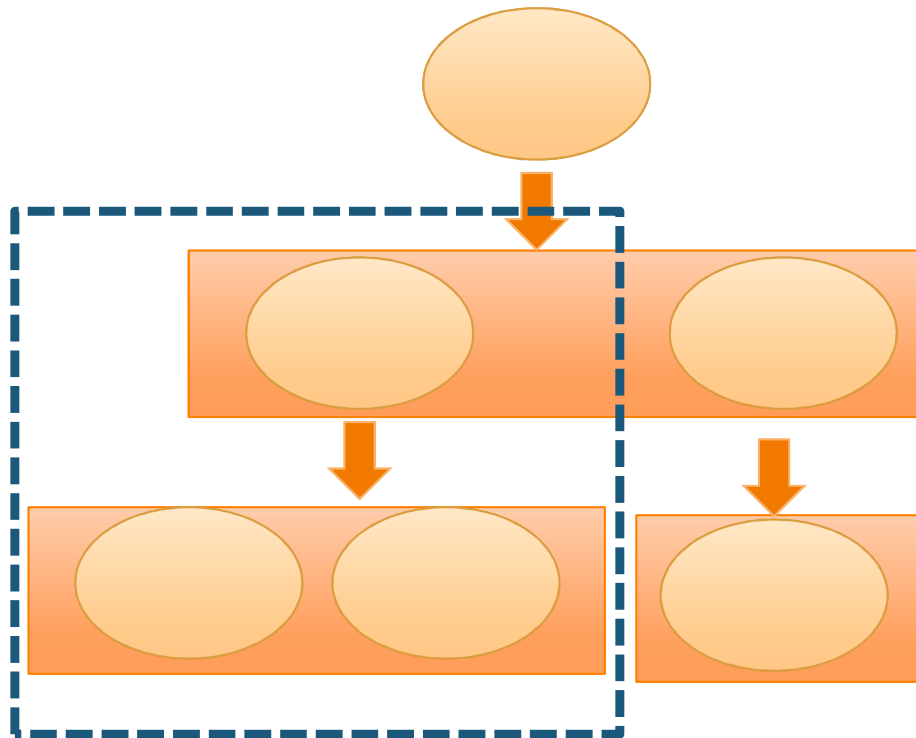
-  The sequential postcondition can state equalities about the whole system.
-  With concurrency, other processes may be operating simultaneously.

# From sequential to concurrent

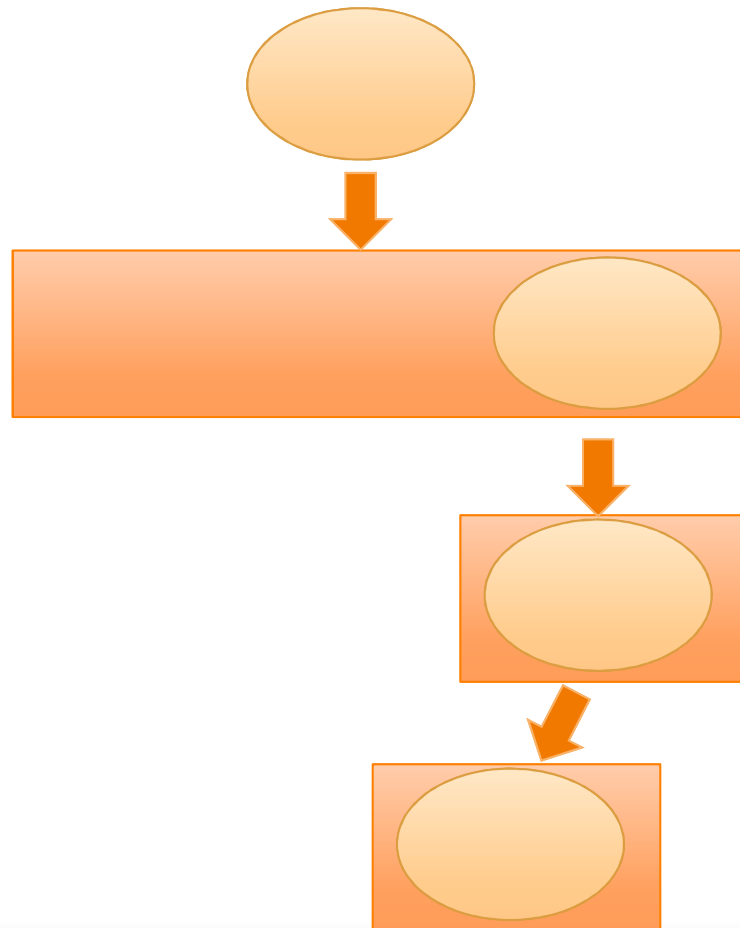
- The sequential postcondition can state equalities about the whole system.
- With concurrency, other processes may be operating simultaneously.
- The postcondition is weakened to form the concurrent postcondition and the guar.



# Abstract Concurrent Remove

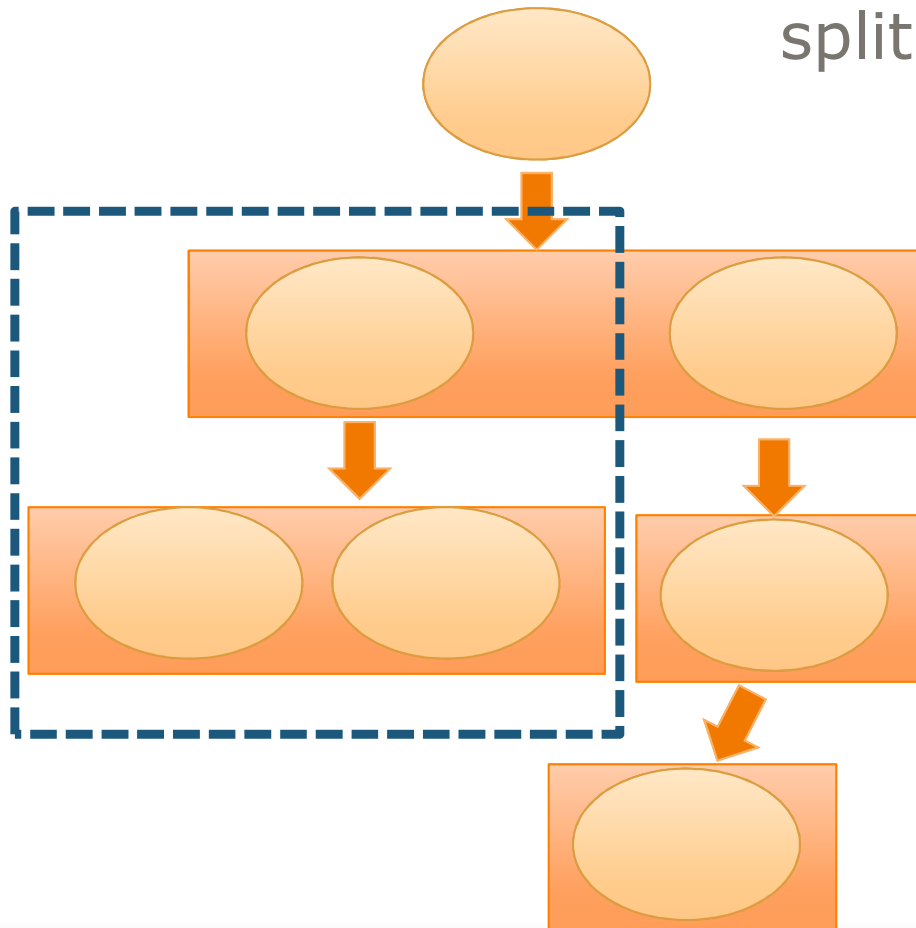


# Abstract Concurrent Remove



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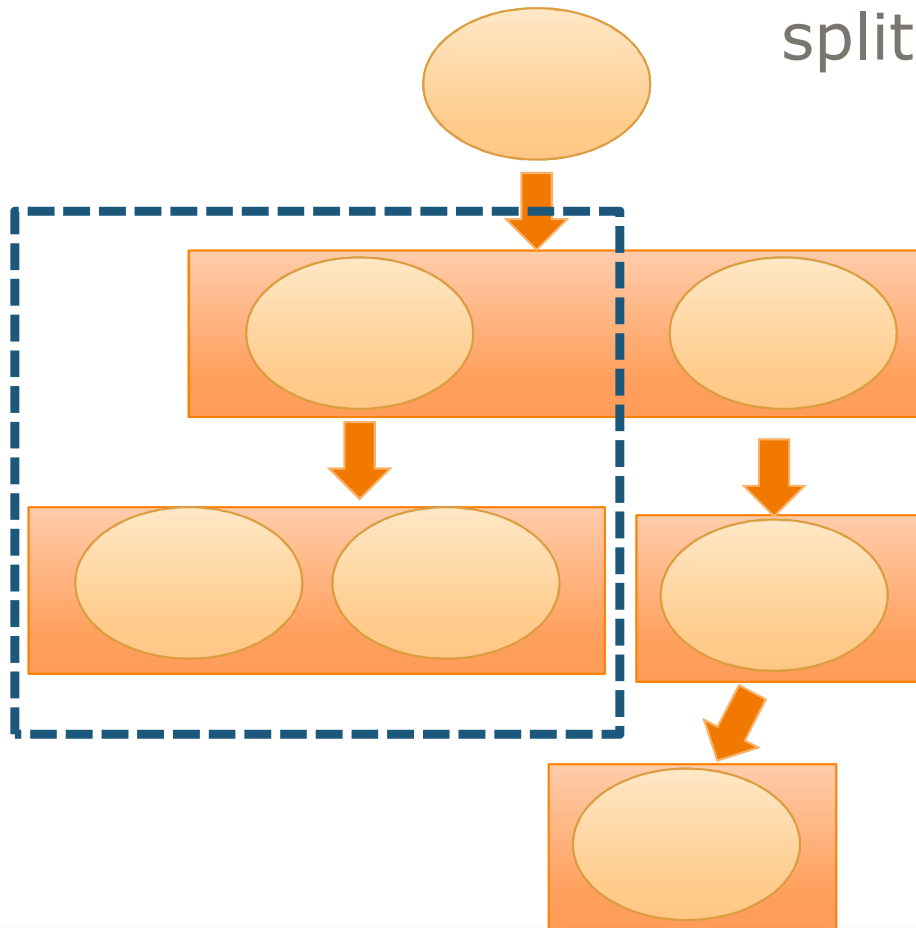
The abstract post is now split into the post and guar:



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post: *child* is removed from *parent's* children list. (Nothing about the rest of the tree).

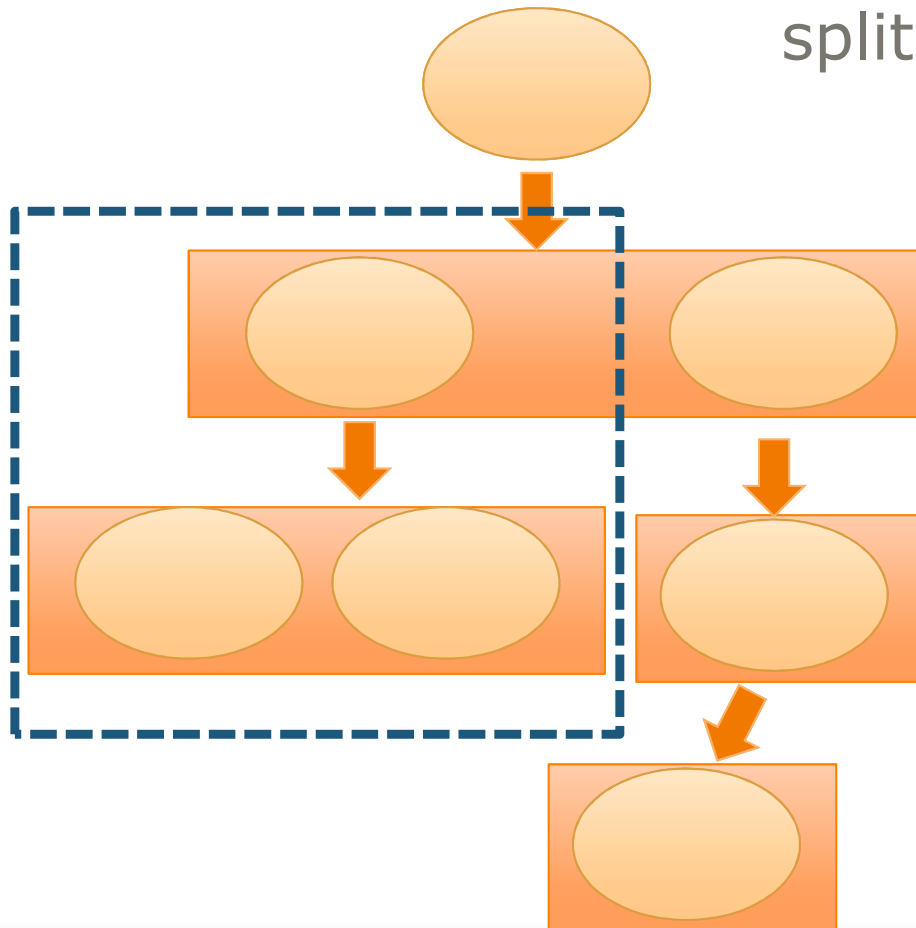


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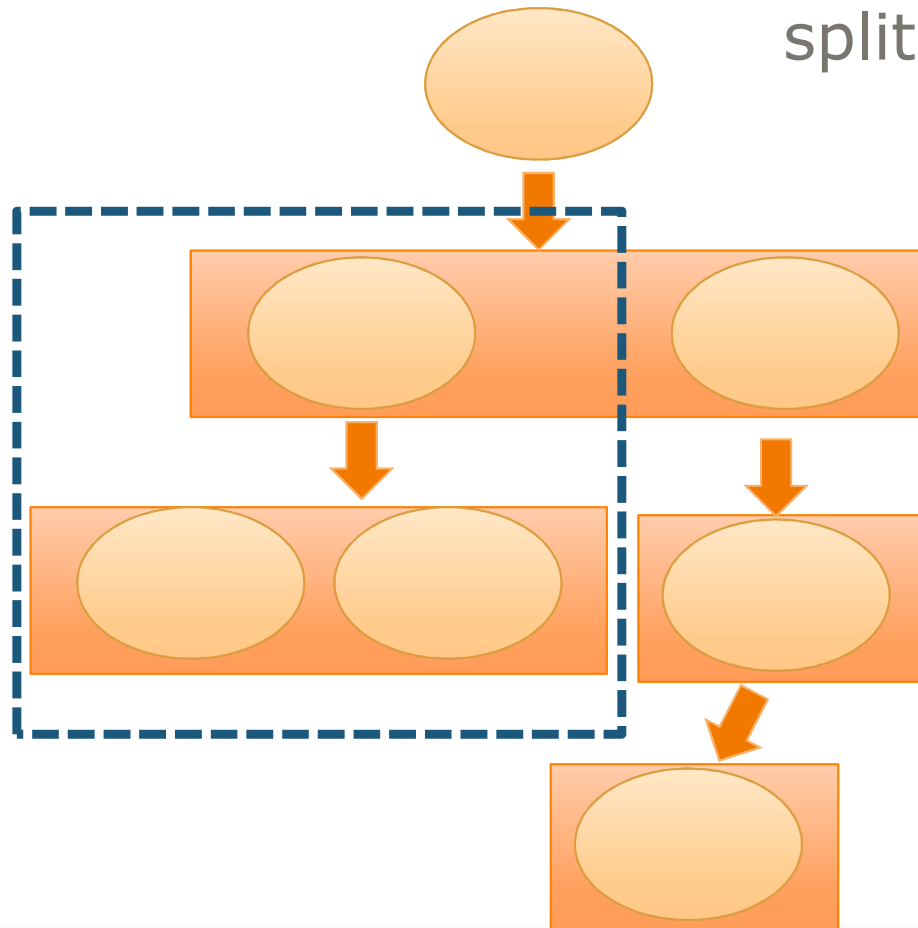
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guar: this process will only change *parent's* children



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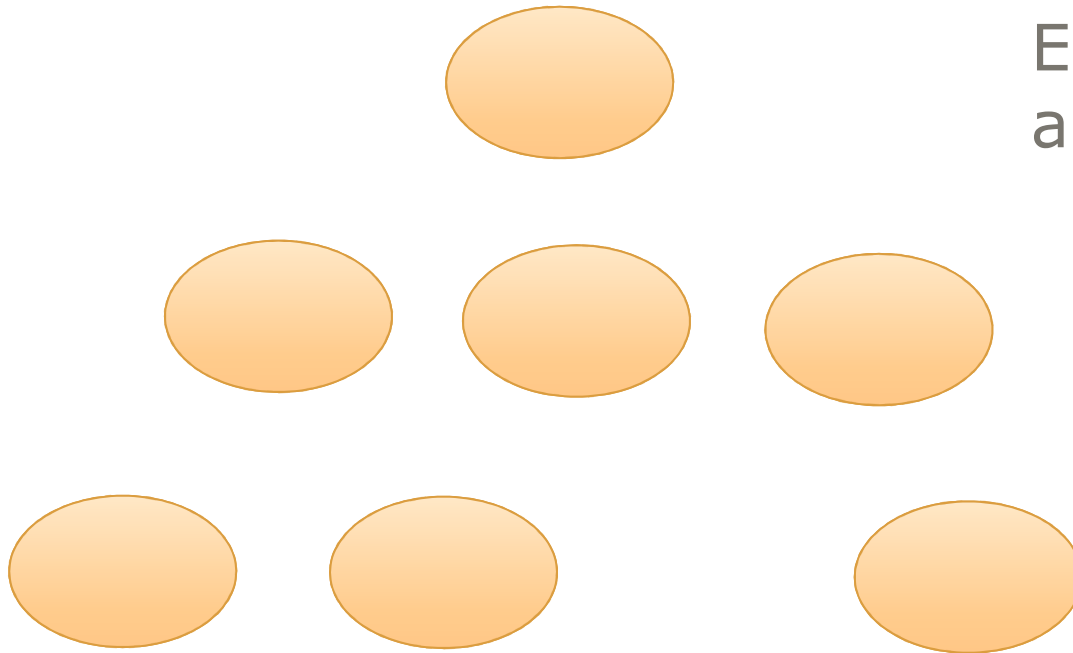
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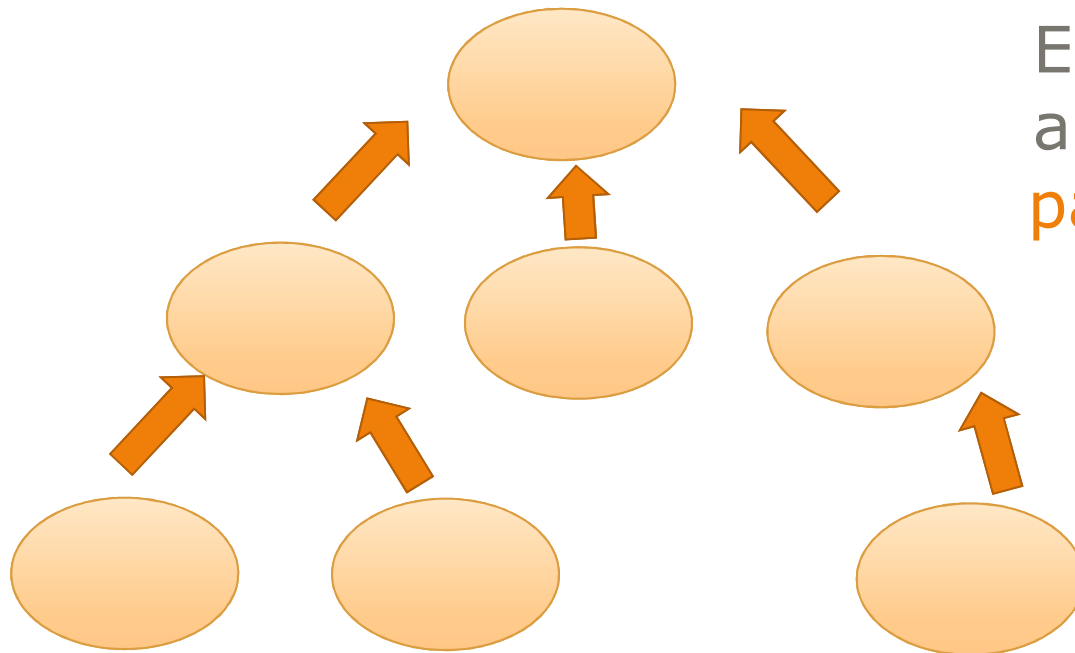
rely: *parent's* children won't change

# Data Reification

Each node has data  
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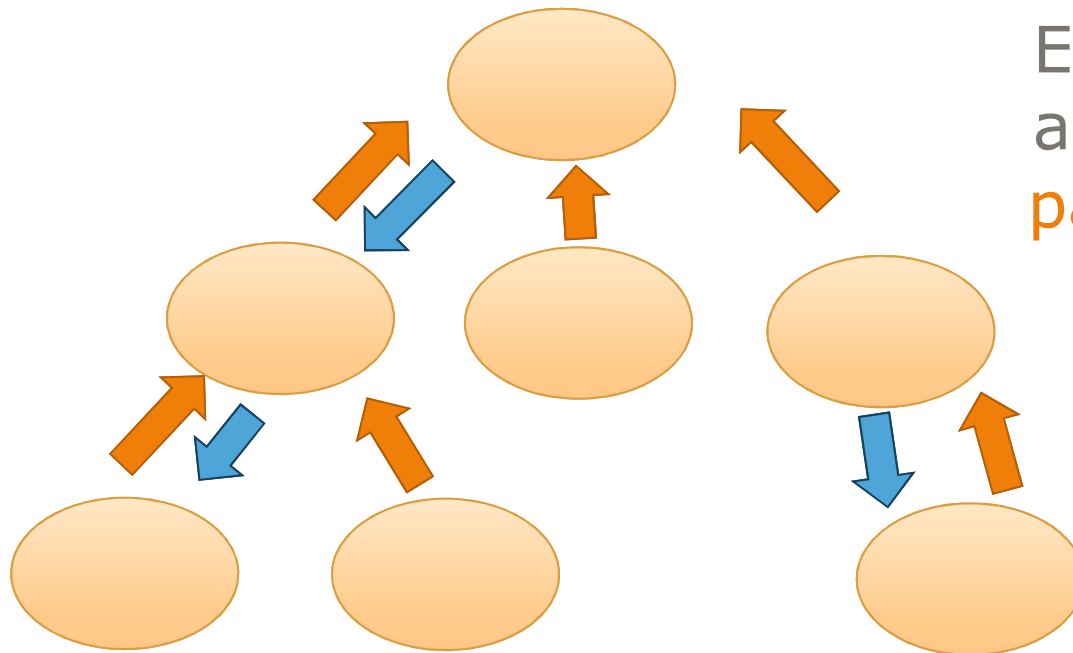
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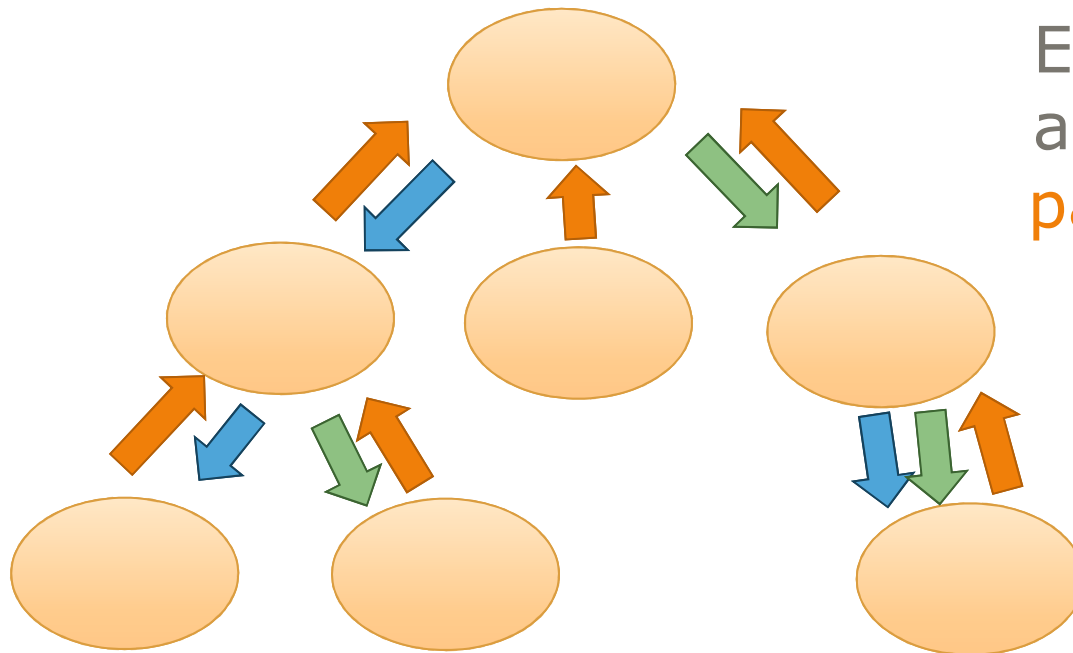


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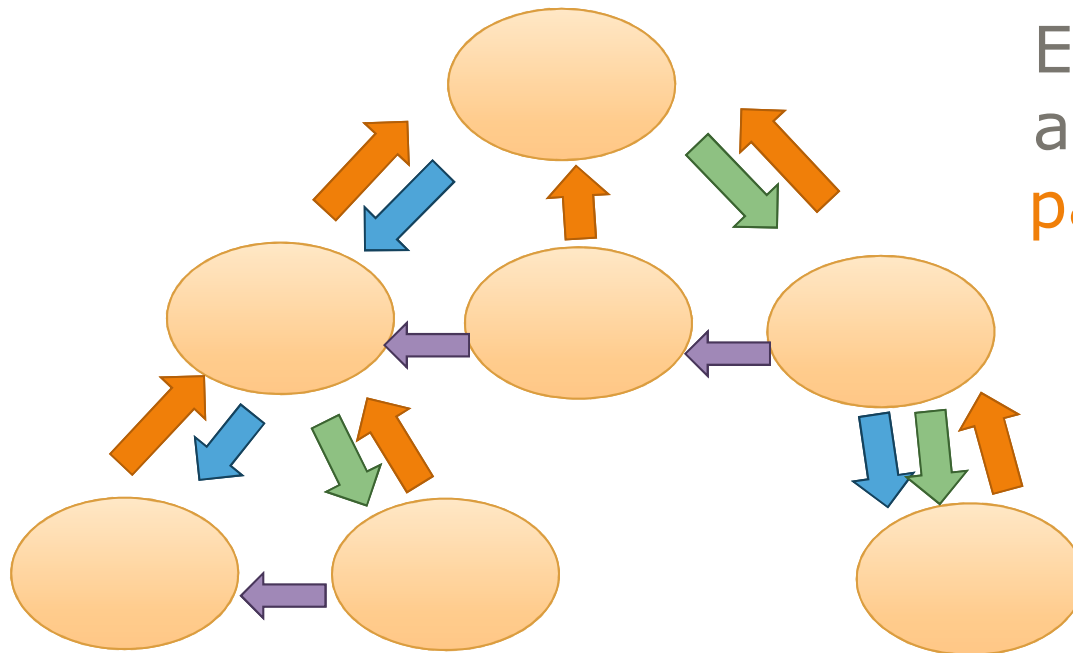
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# Data Reification



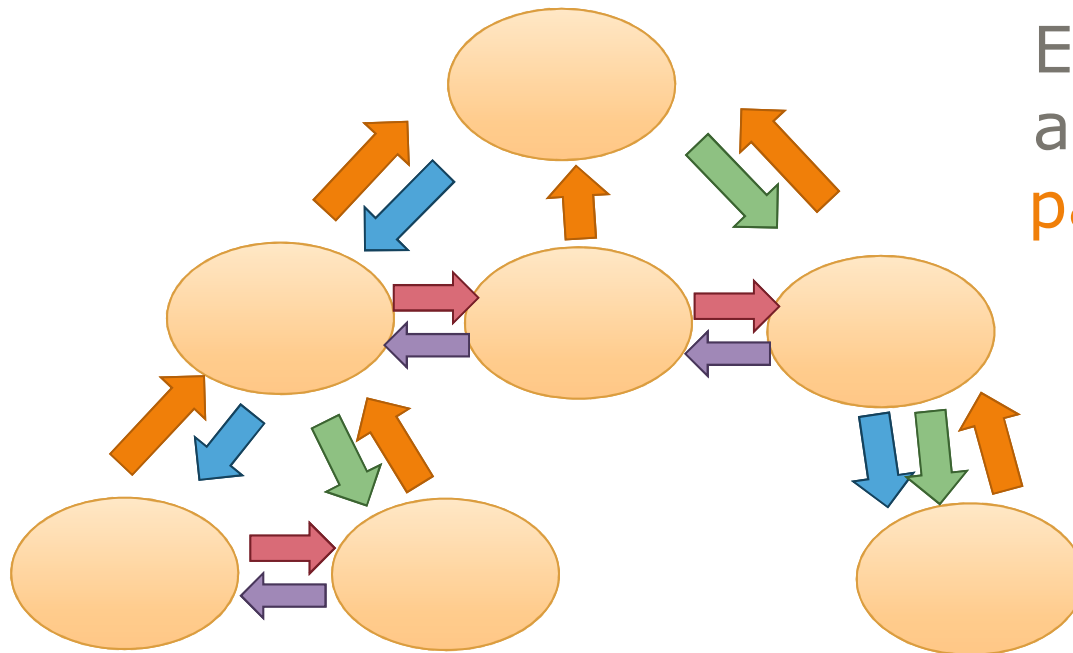
Each node has data  
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# Data Reification



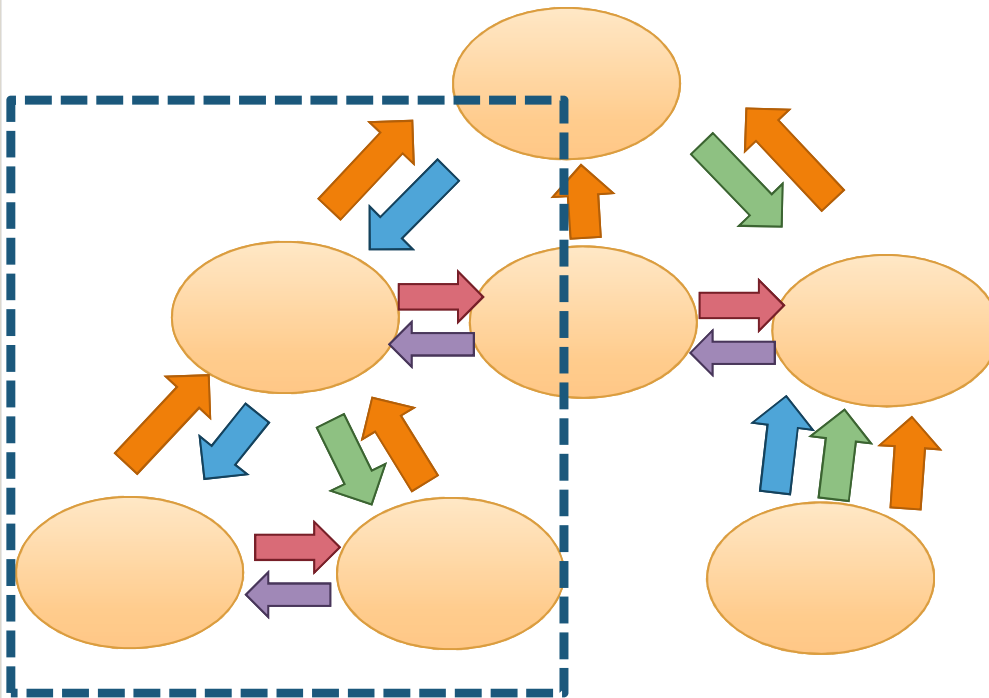
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parent, first child,  
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previous sibling

# Data Reification



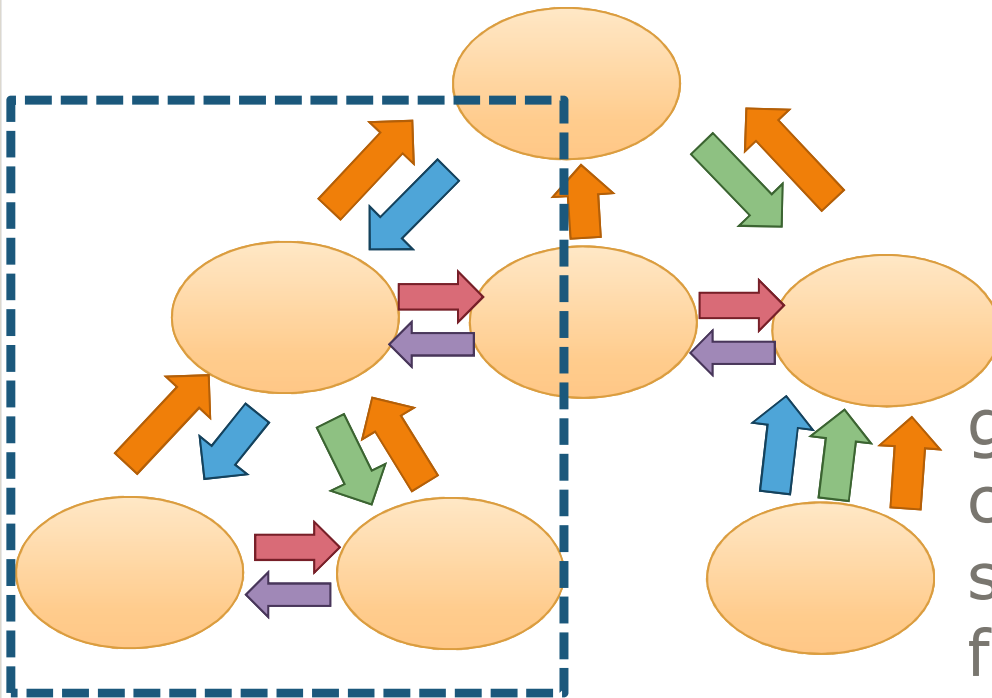
Each node has data and pointers to its  
**parent**, **first child**,  
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**previous sibling**  
and **next sibling**.

# Reified Concurrent Remove



rely: *parent's* children  
won't change, *child's*  
parent & siblings  
won't change.

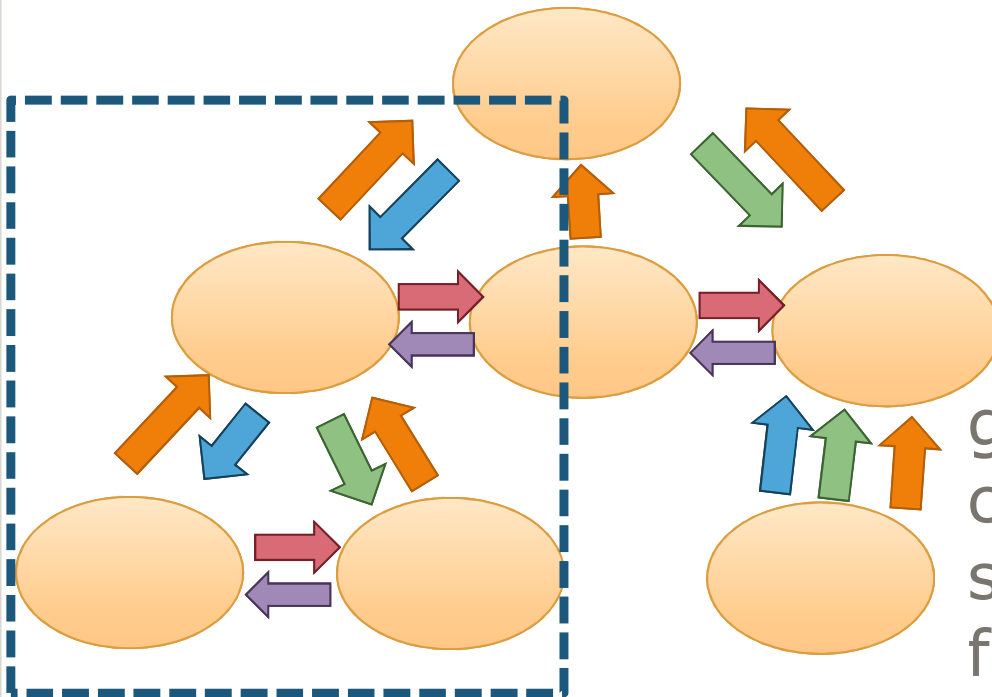
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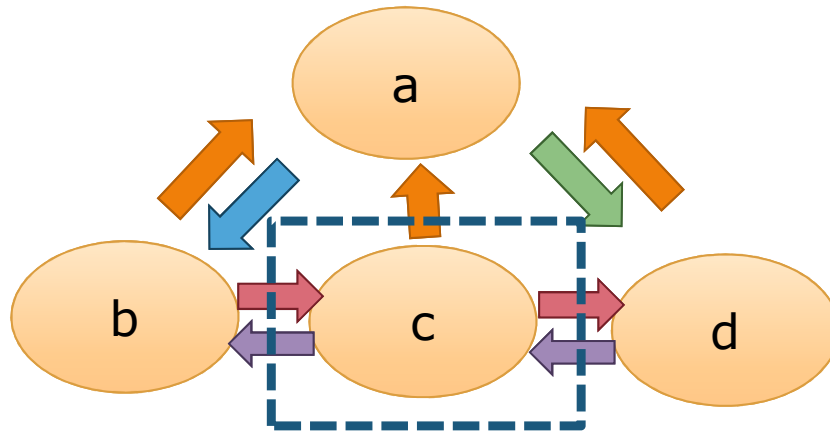


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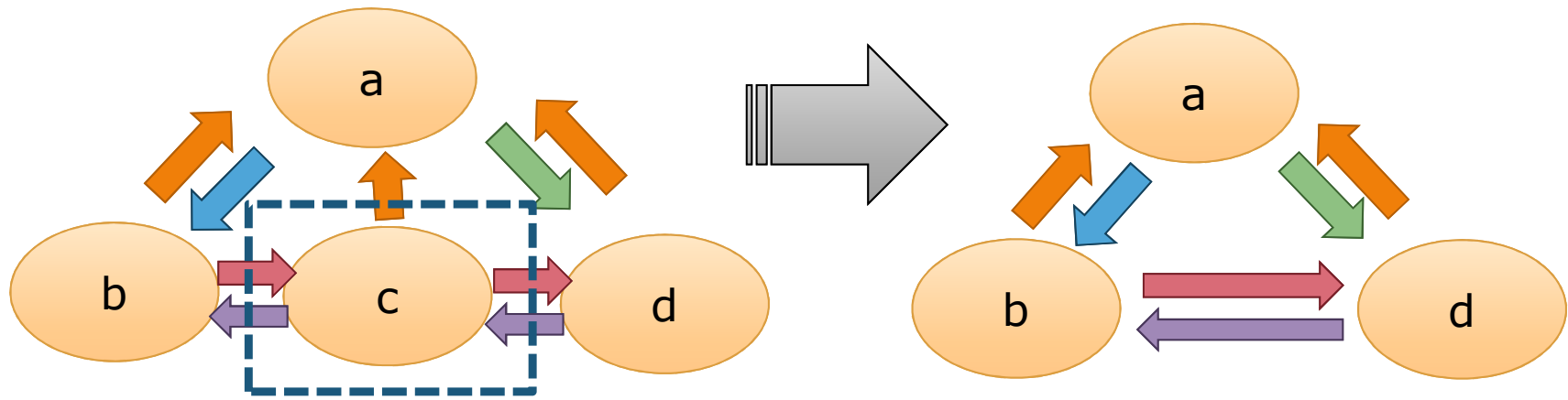
post: *child's* parent, prev & next siblings are nil and the sibling pointers are updated.

# Updating siblings

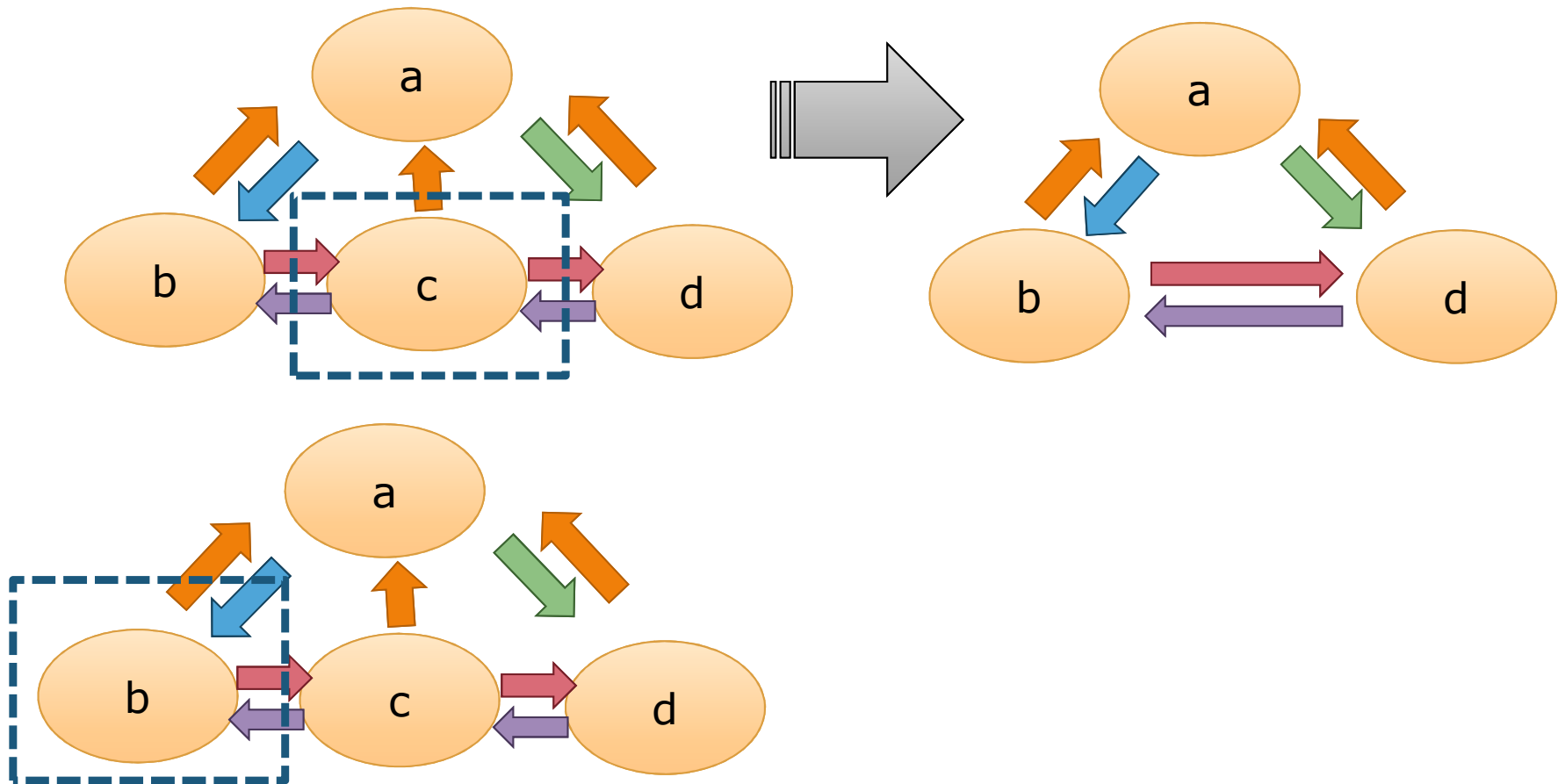




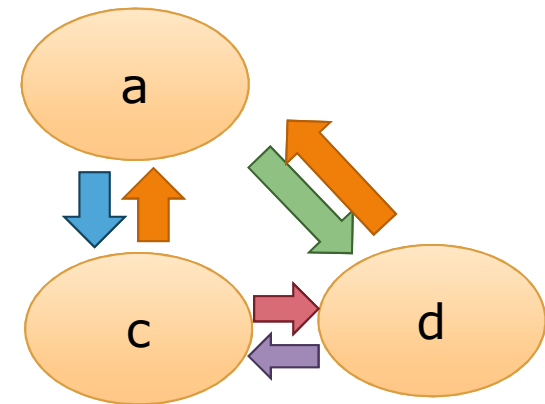
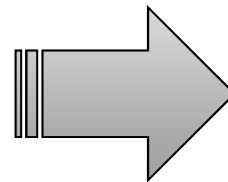
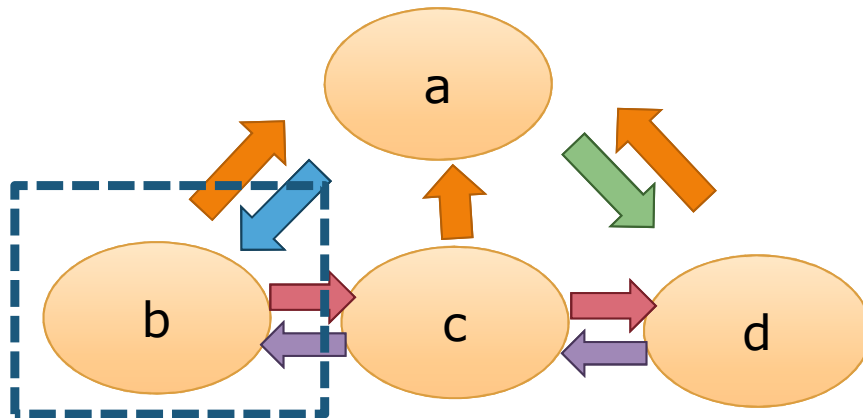
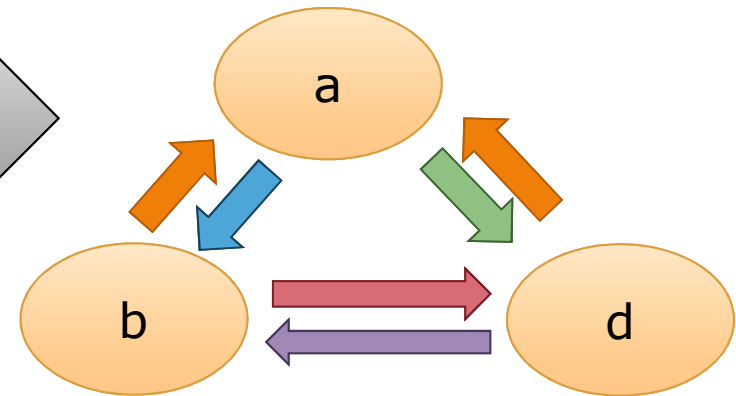
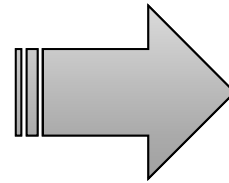
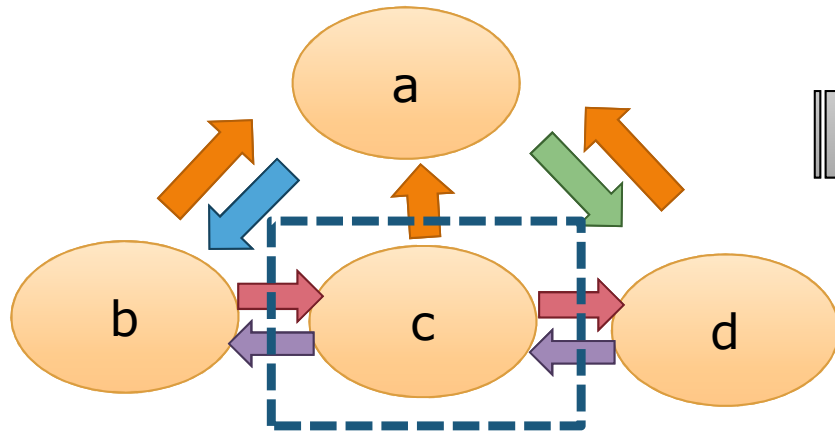
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- For this algorithm, the rely/guar conditions can be satisfied by locking the parent.
- Finer-grained locking can be accomplished by using special sentinel nodes at the corners.

# Conclusions

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- Handling separation by reasoning with layers of abstraction seems promising.
- For concurrency, the sequential postcondition can be weakened to form the concurrent postcondition and the guar condition.
- The process was demonstrated from abstract sequential trees to abstract concurrent trees to reified concurrent trees.